Push-Pull on Graphs is Column- and Row-based SpMV Plus Masks

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Objectives

\begin{itemize}
  \item Investigate the generalizability of direction-optimized BFS.
  \item Investigate how direction-optimized BFS can be expressed using linear algebra.
\end{itemize}

Introduction

Push-pull, also known as direction-optimized breadth-first-search (DOBFS), is a key optimization for making breadth-first-search (BFS) run efficiently. Linear algebra-based frameworks have advantages in conciseness, performance and portability. However, there is no work in literature describing how to implement it within a linear algebra-based framework. Our work shows that DOBFS fits well within the linear algebra-based framework.

Traversal is Matvec

![Graphical representation of BFS traversal](image)

Direction-optimized BFS

Pull

![Pull iteration](image)

Push

![Push iteration](image)

Optimizations

\begin{itemize}
  \item Opt. 1: Column-based masked matvec can use input sparsity.
  \item Opt. 2: Row-based masked matvec can use output sparsity.
  \item Opt. 3: Row-based masked matvec can use output sparsity and early-exit.
\end{itemize}

Complexity Results

<table>
<thead>
<tr>
<th>Operation</th>
<th>Expected Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row- unmasked</td>
<td>$O(dM)$</td>
</tr>
<tr>
<td>Column- unmasked</td>
<td>$O(d \min(m, f) \log M)$</td>
</tr>
<tr>
<td>Row- masked</td>
<td>$O(d \min(m, f) \log M)$</td>
</tr>
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</tr>
</tbody>
</table>

Table: Four sparse matvec variants and their associated cost, measured in terms of number of expected memory accesses into the sparse matrix $A$ required.

Experimental Results

![Comparison of our work to other graph libraries](image)

In this paper we demonstrate that push-pull corresponds to the concept of column- and row-based masked matvec. A possible future research direction would be to extend masking to other applications such as triangle counting and enumeration, adaptive PageRank, batched betweenness centrality, maximal independent set, and convolutional neural networks.

Conclusion

In this paper we demonstrate that push-pull corresponds to the concept of column- and row-based masked matvec. A possible future research direction would be to extend masking to other applications such as triangle counting and enumeration, adaptive PageRank, batched betweenness centrality, maximal independent set, and convolutional neural networks.

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