**Delta-Stepping Synchronous Parallel Model**

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**Abstract**

Many synchronous parallel algorithms like PageRank require a large number of iteration steps. The overheads of global synchronizations on general-purpose cluster take substantial proportion of the execution time for lightweight computations. We propose a variant of Bulk Synchronous Parallel (BSP), Delta-Stepping Synchronous Parallel (DSP), with fewer iteration steps. It achieves faster convergence process by exploring full advantage of data locality.

**Introduction**

Many step-wise parallel algorithms can be intuitively expressed as BSP pattern [2]. This type of parallel pattern is described as in Figure 1.

![Diagram](image)

**Figure 1:** The BSP pattern: One round of local computation is conducted in each substep.

As is shown in Figure 1, varying degrees of interdependence exist among these processors \{P_i, P_j, \ldots, P_n\}. When the dependence between \(P_i\) and \(P_j\) \((j \neq i)\) is subtle, the orientation of the convergence of \(P_i\) will be mainly decided by the data residing in itself. So we conjecture that increasing local computing steps in each substep will speed up local convergence, sequentially advance the global convergence. The idea is sketched in Figure 2.

By further formalization and derivation, we prove that, if the algorithm converges with two local computations, then it converges with any number of local computations. For the convex optimization problems and local-optimal insensitive problems, the convergence is sufficient.

**Case Study**

To demonstrate the applicability and performance, we apply the model on several algorithms: Max Value Propagation (MVP), Jacobi Iterative Method (JIM), Single Source Shortest Path (SSSP) and PageRank (PR).

As is shown in Figure 3, the figures show that DSP reduces the numbers of iterations and communication of MVP, JIM, SSSP and PR significantly. Figure 4 shows that DSP reduces the execution time and the number of iterations of SSSP and PageRank dramatically.

**Conclusion**

DSP is a variant of BSP. It utilizes inaccurate global data when performs multiple computation steps in each substep. These advanced computation steps further exploit the locality of data, and accelerate the convergence. This research was supported by the Zhejiang Engineering Research Center of Intelligent Medicine (2016C10011) and the research and application of key technologies for rapid individualized sculpture manufacture and carving stone materials appraisal (2016C03SAB80611).

**References**


**Figure 2:** The DSP pattern: \( \Delta \) rounds of local computation is conducted in each substep.

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**Figure 3:** Performance comparison between DSP and BSP. The graphs used in (a, c, d) are random graphs, \( p_{in}, p_{out} \) indicate the possibilities of a edge existed between a pair of vertices in the same partition and different partitions respectively.

**Figure 4:** Performance comparison between DSP and BSP. The graphs used in (a, c, d) show the results of SSSP working on well-partitioned and random-partitioned subgraphs respectively. The convergence of MVP, JIM, SSSP and PR significantly. Fig-